The Effect of Fibre Length and Interfacial Bond in Glass Fibre-Epoxy Resin Composites

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The effect of fibre length on the strength of glass fibre-epoxy resin composites has been examined by beam bending experiments on uniaxially aligned material. The results agree well with theoretical predictions and the critical fibre length is found to be 12.7 mm (0.5 in.).

A method of measuring the interfacial shear strength of the fibre-matrix interface has been developed and the measured value of the interfacial shear strength found to be 9.5 N mm^{-2}.

The mechanism of shear failure is examined and discussed in detail.

1. Introduction

Glass fibre laminates generally show a marked loss of strength when the fibre length falls below ~13 mm [1] (0.5 in.). Kelly and Tyson [2] have deduced a series of equations relating fibre length to the strength of composites and have shown that the strength of the composite σ_c can be related to fibre length l by the following equation

$$\sigma_c = \sigma_f V_f \left(1 - \frac{l_c}{2l}\right) + \sigma_m(1 - V_f) \qquad (1)$$

where σ_f is the tensile strength of the fibre, σ_m is the tensile strength of the matrix, V_f is the volume fraction of fibre and l_c is a critical fibre length, below which the fibre is pulled out of the matrix before developing its full strength. Recently, Carswell and Lockhart [3], using uniaxial tensile tests on randomly oriented glass fibre, with both epoxy and polyester resin matrices, have also found that the critical length is of the order of half an inch (~13 mm). If load is transferred from the matrix to the fibre by shear stresses at the fibre-matrix interface and this interfacial shear stress is τ_i, then a simple force balance shows that the critical length of a fibre of diameter d is given by:

$$\frac{l_c}{d} = \frac{\sigma_f}{2\tau_i} \; . \qquad (2)$$

Substituting typical values for l_c (13 mm), and d (13 μm) makes the ratio l_c/d, termed the critical aspect ratio, approximately 1000. However, the strength of fibre glass used in fibre laminates is of the order of 1750 N mm^{-2} and values of the interfacial bond, measured by pulling single fibres from a matrix [4], are approximately 20.6 N mm^{-2}, which suggests that the value of the critical aspect ratio should only be about 43. There is a very obvious discrepancy in these values and the present work was undertaken to determine whether, for a simple uniaxially aligned system of glass fibre in an epoxy matrix, experimental results would agree with theoretical predictions and in particular, whether the critical length and interfacial shear strength could be measured and related.

2. Beam Bending Experiments

The influence of fibre length on the strength of glass fibre composites was examined by studying the deformation of fibre-reinforced beams under a modified three-point loading system. The materials used were glass fibre-reinforced epoxy resin composites. The fibres were in the form of rovings which consisted of 32 ends, each containing approximately 140 fibres of diameter 12.7 μm (5 $\times$ 10^{-4} in.). The resin used was Araldite 219 containing 100 parts Araldite 219,

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50 parts Hardener 219 and 3 parts Accelerator 219, the accelerator being added immediately before casting to avoid premature reaction taking place. Rectangular beam specimens were made of length 127 mm, breadth 6.35 mm and depth 4.76 mm. The specimens were made by "hand laying" in a specially designed collapsible jig which was previously coated with a release agent. The jig contained six channels, each of length 305 mm, from which twelve, 127 mm individual specimens could be made and cured together. After curing the end 25 mm were cut from each length of material, to minimise end effects and the middle section was used to prepare two identical 127 mm specimens. In this way twelve individual specimens could be made and cured together.

The volume fraction of glass fibre was determined by weight and was kept constant in all specimens at 26 ($\pm$1)%. Fibre lengths of 127 mm (continuous fibre), 50.8 mm, 25.4 mm, 17.5 mm and 9.52 mm were used and one batch of resin beams were made without fibre additions. The fibres were cut to the required length and then vibrated to effect separation. They were then fed into the 305 mm moulds in the jig via a trough with a slit along the bottom which allowed the fibres to fall in one direction only, care being taken to ensure random distribution. The usual practice of hand laying was used, with alternate additions of resin and fibre until the mould was full, the fibres being compressed into the resin by means of a small hand roller after each addition.

All the specimens were given a standard curing treatment of $1\frac{1}{2}$ h at 100° C in a thermostatically controlled oven followed by 24 h ageing at room temperature prior to testing.

The specimens were tested using a standard three-point bending rig designed for use with a Hounsfield testing machine but trouble was experienced by the centre loading pivot causing localised damage to the specimens. Pivots of various radii were made to eradicate this defect but the problem was finally overcome by placing a piece of thin spring steel between the centre pivot and the specimen. This eliminated the notching effect, and the force required to bend the thin steel was negligible compared to the force required to deform the composites. The interface between the steel and composite was greased with silicone grease to minimise frictional effects but this proved to be unnecessary.

Six bending tests were made for each length of fibre, and the force to cause failure was measured. The maximum stress in the beam was computed from simple beam theory.

$$\sigma = \frac{3PL}{2bh^2} \qquad (3)$$

where σ is the maximum stress in the beam; P is the force recorded; b is the breadth of the bar and h is the depth. L, the distance between the outer supports, was 60 mm.

The results of these tests are given in table I and the effect of fibre length on the strength of the composite is shown in fig. 1.

The strength of a composite made from discontinuous fibres can be expressed in terms of the strengths of the fibres and matrix by equation 1 developed by Kelly and Tyson [2]. This relationship assumes that the load on the fibre builds up linearly from the ends over a distance of $l_c/2$ and then reaches the critical value σ_f which is constant over the mid-section of the fibre. Whilst this equation is reasonable for composites containing fibres of length greater than the critical value, when the fibre length is smaller than l_c, the critical stress in the fibre (σ_f) is not reached and the average stress in the fibre will be $\frac{1}{2}\sigma_f V_f \; l/l_c$. Equation 1 must therefore be modified as shown below.

$$\sigma_c = \tfrac{1}{2}\sigma_f V_f \; l/l_c + \sigma_m(1 - V_f) \,. \qquad (4)$$

From equations 1 and 4 the value of the strength of the composite (σ_c) could be predicted

TABLE I Beam tests on fibre-reinforced composites of different fibre length

Length of fibre (mm)	Force to failure (Newtons) 1	2	3	4	5	6	Average	Strength of composite (N mm^{-2})
127	800	810	800	792	912	824	823	515
50·8	759	795	768	765	736	780	752	472
25·4	642	714	617	678	642	—	659	413
17·5	578	636	596	578	511	556	579	362
9·52	394	429	420	420	411	429	417	262
Resin alone	151	141	144	153	143	—	146	91·5

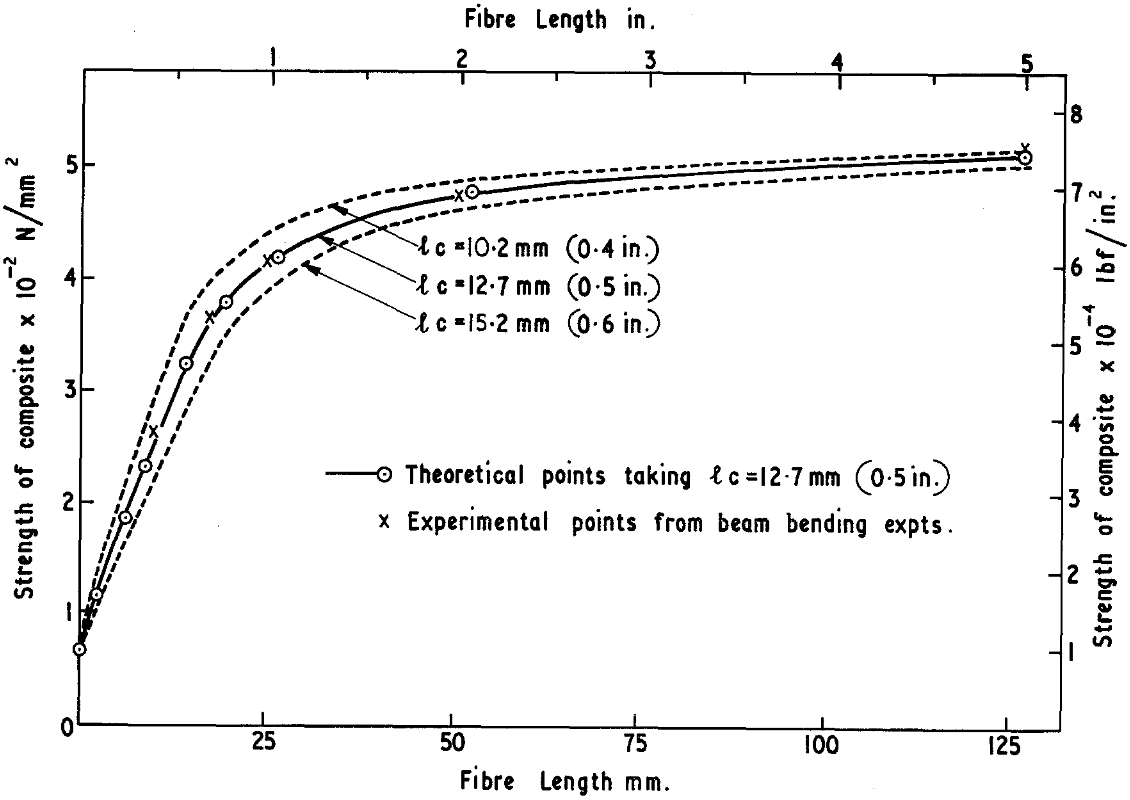

Figure 1 Effect of fibre length on the strength of the fibre-reinforced composites.

if the values of the strengths of the matrix (σ_m) and fibres (σ_f) were known. These values were found as described below.

A value of σ_m was obtained from results of bend tests on the unreinforced matrix and gave 91.5 N mm^{-2} (table I). To confirm this figure uniaxial compression tests were made on the matrix, and the yield point measured. The results, shown in table II, gave an average value of 95 N mm^{-2} which, considering the nature of the tests involved, is in good agreement with the results of the beam tests.

The value of the tensile strength of the fibre σ_f was deduced from the beam experiments with continuous fibres, using the relationship;

$$\sigma_c = \sigma_f V_f + \sigma_m(1 - V_f)\,, \tag{5}$$

and taking the average value of σ_m to be 93.25 N mm^{-2}. This gives a value of σ_f of 1720 N mm^{-2}.

The value of σ_f was confirmed by making uniaxial tensile tests of material containing continuous fibres. The cross-sectional area of the gauge length of these specimens had to be very small to ensure that the specimen failed in

TABLE II Uniaxial compression tests on unreinforced matrix specimen, length approximately 12.7 mm

Specimen width (mm)	Specimen thickness (mm)	Force to yield (N)	Matrix yield stress σ_m (N mm^{-2})
5·26	6·25	3200	97·0
5·29	6·25	3380	102·0
5·55	6·20	3200	93·0
5·70	6·25	3210	90·1
5·24	6·20	3020	93·0
5·57	6·35	3380	95·7
5·42	6·25	3150	93·0
5·24	6·25	3110	95·0
		Average	95·0

TABLE III Uniaxial tensile test results on continuous-fibre reinforced material

Test no.	Cross-section area (mm^2)	Force to fracture (N)	Tensile strength σ_c (N mm^{-2})	Tensile strength of fibre σ_f (N mm^{-2})
1	3·02	1495	495	1640
2	1·57	957	610	2080
3	1·57	780	496	1640
			Average	1790

tension across the fibres, rather than by shear or crushing in the grips. Satisfactory specimens were made from beam specimens by "waisting" them using a 25.4 mm radius cut to give a tensile cross-section of approximately 1.5 to 3 mm^2. Each specimen was accurately measured and the tensile strength of the fibres σ_f computed from equation 5 using the previously determined value of σ_m (93.25 N mm^{-2}). The results of these tensile tests are given in table III. An average value of σ_f of 1790 N mm^{-2} was obtained which agrees well with the value of 1720 N mm^{-2} obtained from the earlier bend tests.

Using the above value of σ_m (93.25 N mm^{-2}) and an average value of σ_f of (1755 N mm^{-2}), the strength of the composite σ_c was calculated from equation 1 (for fibres above the critical length) and from equation 4 (for fibres below the critical length). In fig. 1 the calculated values of σ_c are shown when critical lengths of 10.2, 12.7 and 15.2 mm are taken. It can be seen that the experimental results agree almost exactly with theoretical predictions when the critical length is taken as 12.7 mm. This value is in agreement with the recent work by Carswell and Lockhart [3] at the National Engineering Laboratory.

3. Measurement of the Interface Bond τ_i

Taking the measured values of l_c, σ_f and d from the previous section, equation 2 predicts that τ_i should be $\sim$0.9 N mm^{-2}. To determine the value of τ_i experimentally, specimens of the form shown in fig. 2 were made from material containing continuous fibres. The geometry of the specimens is such that the specimens fail by shear along the broken lines shown and the true failure area was obtained by careful measurement of the fracture lengths and cross-sections using a projection microscope at a magnification of $\times$ 1000. This enabled the actual fracture cross-section to be measured accurately; the measurement included the length of fracture between fibres and the distance traversed around each fibre. From this data the average shear stress of the composite τ_c was determined and the results are given in table IV.

A value of the interface bond τ_i can now be obtained from the equation

$$\tau_c = x\tau_i + (1 - x)\tau_m \qquad (6)$$

where x is the fraction of the fracture area occupied by the fibre interface and τ_m is the matrix shear strength. The value of τ_m can be obtained from the earlier tests for, as the value of τ_m in tension (91.5 N mm^{-2} from beam tests) is similar to the value obtained in compression (95 N mm^{-2} table II), it is reasonable to assume that the mechanism of failure is by shear in both cases. τ_m should therefore be equal to half the value of σ_m. Taking the average value of σ_m from the beam and compression results gives a value of τ_m of 46.6 N mm^{-2}. It remains only to determine the percentage of fibre-matrix interface length in the fracture surface.

Assuming that the fibres are dispersed uni-

TABLE IV Shear tests on continuous-fibre reinforced materials

Specimen thickness (mm)	Average width of actual fracture surfaces measured on microscope (mm)	Total length fracture (mm)	Actual fracture area (mm^2)	Force to fracture (N)	Average shear stress τ_c (N mm^{-2})	τ_i assuming a square array distribution (N mm^{-2})
3·46	4·40	20·8	91·8	1900	20·8	8·7
3·56	4·54	33·2	150	3450	23·0	11·9
3·30	4·20	23·4	98·3	2490	25·4	15·4
2·21	2·80	22·2	62·2	1480	23·8	13·1
				Average	23·2	12·3

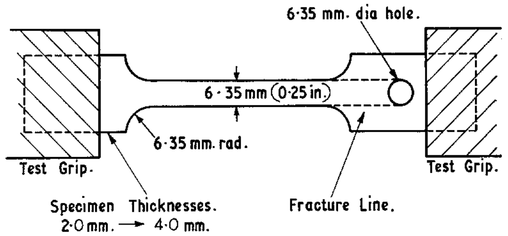

Figure 2 Test specimen used for determining the shear strength of the composites.

formly in a square array throughout the cross-section of the specimen, and that on average the crack travels half way round the fibres before proceeding, then, as shown in the appendix, the percentage of fibre-matrix interface x in the fracture surface turns out to be 68% for a fibre volume fraction of 0.26. Using this value in equation 6 gives a value of τ_i of 12.3 N mm^{-2} (table IV), which is fourteen times larger than predicted from beam bending data.

If this value of τ_i, 12.3 N mm^{-2}, is substituted in equation 2 an aspect ratio of about 71 is obtained. This could be explained by assuming that the fibres are bunching together to give an apparent fibre diameter of 178 μm, i.e. that the fibres are behaving as bunches with approximately fourteen fibres across a bunch. Microscopical examination of the composites showed that the fibres, were not ideally dispersed but did tend to gather in bunches, as shown in fig. 3.

The calculation of τ_i assumes that the fracture path runs straight through an idealised square array of fibres, but, as the values of the interfacial shear strength and matrix shear strength are appreciably different, this might be an over-simplification. If there were more fibres in the fracture area than expected, then the value of the interfacial shear strength, calculated from equation 6, would change. The influence of increasing the number of fibres in the fracture surface is shown in fig. 4.

To check whether the fracture ran preferenti-

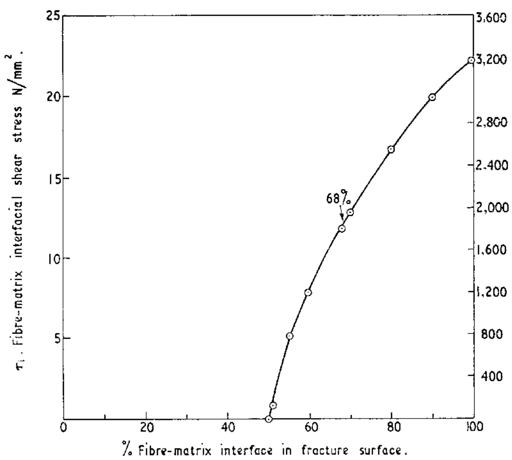

Figure 4 The effect of the amount of fibre-matrix interface in the fracture surface on the calculated interfacial shear stress.

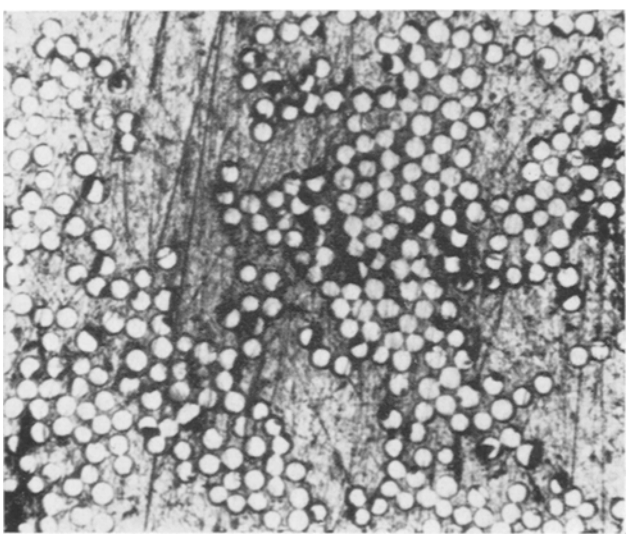

Figure 3 Distribution of fibres in composites (× 200).

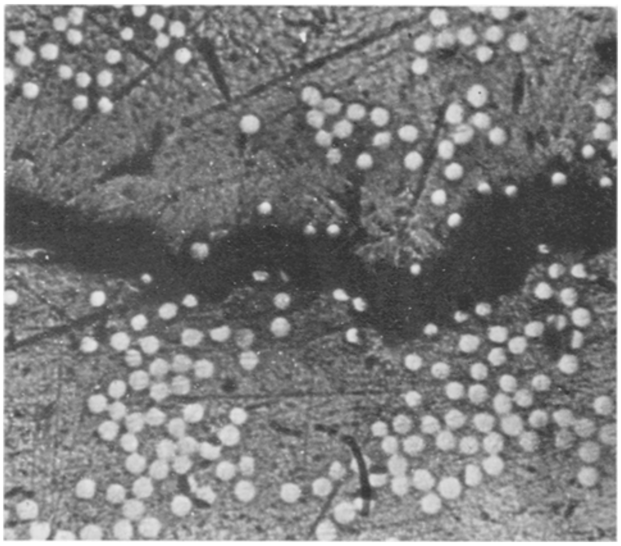

Figure 5 Fracture path in shear failure test ($\times$ 200).

ally through the matrix the two halves of the shear specimens were bound together after fracture, and mounted in perspex. The mount was sectioned and polished to reveal the fracture path, fig. 5. The fracture was projected using a projection microscope as before at a magnification of $\times$ 1000 and the whole fracture surface was traced accurately on a continuous roll of transparent plastic, the position of the fibres (counting those on both sides of the fracture) were marked. At this magnification the projected fracture surface measured over 2.5 m and the length could be measured accurately, including the path around the fibres, using an instrument for measuring distances on geographical maps. Measurement of a fractured surface gave a value of the number of fibres per unit length of crack of 311 fibres cm^{-1}. A second similarly prepared specimen gave a value of 320 fibres cm^{-1}. If the average of these two results is taken, i.e. 315 fibres cm^{-1} then, following the appendix, this is equivalent to a value of the percentage fibre interface x of 63% which means that τ_i is 9.5 N mm^{-2}.

Therefore the assumption that the fracture runs straight through a square array is only in error by 25%. However, these results show that the total percentage of fibre-matrix interface in the fracture surface is smaller than would be expected by assuming a reasonable dispersion of fibres in the fracture path. This means that the fracture runs preferentially through the matrix, in spite of the fact that the interface strength is very low and this is similar to Cook and Gordon's [5] observation that a fibre interface can act as a very effective crack arrester. In this case if the fracture is considered to spread from left to right in fig. 6 then, as it encounters the first fibre and runs half-way around as shown, the crack front is now pointing at right angles to the intended linear progression and this would tend to divert the fracture path and increase the energy required for fracture.

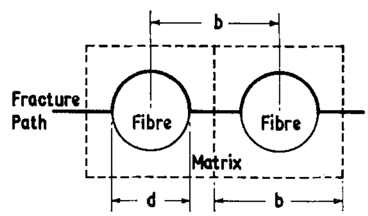

Figure 6 Idealised shear fracture path in composites containing perfectly distributed fibres.

One final point is that the value of τ_i obtained (9.5 N mm^{-2}) is lower than data previously published from results of experiments where single fibres were embedded in a resin matrix (20.6 N mm^{-2}). This may be due to differences in surface condition of the fibres, but a reduction in the interfacial shear stress is expected as the fibres approach one another because, if the load is transferred to the fibres mainly by friction, as suggested by Outwater [6], equation 2 can be rewritten as:

$$\frac{l_c}{d} = \frac{\sigma_f}{2\mu F(t/d)} \qquad (7)$$

where t is the distance between fibres, μ is the coefficient of friction and F is the normal stress on the fibre due to contraction of the resin. The apparent interface shear stress will reduce as t decreases and therefore when the fibres pack closely it is to be expected that the interface bond should be smaller in composites than in a matrix containing a single fibre, and this is exactly what is observed.

4. Conclusions

The effect of fibre length on uniaxial glass fibre-epoxy resin composites was examined and found to agree very well with theoretical predictions.

The critical fibre length was measured from beam bending experiments and found to be approximately 12.7 mm (0.5 in.), which agrees with other recent work using tensile tests on randomly aligned fibres [3].

A method of measuring the fibre-matrix interfacial shear strength has been developed and, assuming an ideal square array of fibres, a value of the apparent fibre-matrix interfacial shear strength of 12.3 N mm^{-2} was obtained. Investigation of the fracture mechanism showed that the fracture runs preferentially through the epoxy matrix and that the actual value of the interfacial shear strength is 9.5 N mm^{-2}.

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Appendix

The proportion of Fibre-Matrix Interface in the Shear Fracture Surface Assuming an Idealised Fracture Path

The following analysis assumes the fibres are dispersed in a uniform square array and that the fracture runs as shown in fig. 6.

If d is the fibre diameter and b the distance between fibre centres, then the volume fraction V_f is given by

$$V_f = \left(\frac{\pi d^2}{4}\right)(b^2)^{-1}. \qquad (A1)$$

The percentage of fibre-matrix interface in the fracture surface x is

$$x = \frac{\pi d/2}{(b-d) + \pi d/2} \times 100\% \qquad (A2)$$

Substituting for b from equation A1 gives

$$x = \frac{\sqrt{(V_f \pi)}}{\sqrt{(V_f \pi)} + 1 - 2\sqrt{(V_f/\pi)}} \times 100\%$$

If $V_f = 0.26$ then $x = 68\%$. Thus ideally the percentage of fibre-matrix interface in the fracture surface should be 68 % when the volume fraction of fibres is 0.26.

References

1. K. J. BROOKFIELD, in "Fibre Reinforced Plastics", edited by P. Morgan (Iliffe, New York, 1954).
2. A. KELLY and W. R. TYSON, Chapter 13 in "High-Strength Materials", edited by V. F. Zackay (John Wiley and Son, New York, 1965), p. 587.
3. W. S. CARSWELL and A. H. LOCKHART, "The Effect of Fibre Length on the Tensile Strength of Glass Resin Composites. (Internal Report, National Engineering Laboratory).
4. R. H. NORMAN, D. I. JAMES, and G. M. GALE, *Chem. Eng.* (October 1964) 243.
5. J. COOK and J. E. GORDON, *Proc. Roy. Soc.* **A242** (1964) 508.
6. J. O. OUTWATER, JR, *Modern Plastics* **33** (1965) 156.

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